Scientific Reasoning and Belief Structure

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Abstract *For explaining theory change, it is important to be able to express which theory is believed to be fundamental and which theory is thought to be true. To achieve this expressive power, I propose theory of belief structures (TBS) that describes belief states not in terms of a set of sentences but by a structure. The dominant formal approach of belief revision, the AGM theory, adopts a conservative strategy. However, this does not always conform to the praxes of scientists. Contrarily, TBS admits to taking a progressive strategy and can describe different kinds of belief states. To demonstrate the descriptive power of TBS, it is applied to a description of change in astronomical theory.*

Keywords: scientific reasoning, belief structure, theory change, belief revision, progressive strategy, scientific revolution.

1 Theory of Belief Structures

In this paper, *theory of belief structures* (TBS) is proposed. TBS treats a bundle of statements as a unit of belief and takes evaluations of beliefs into consideration. TBS expresses an epistemic state not through a set of sentences but through a structure constructed from a set of sets of sentences and an ordering among them. An early version of TBS was proposed in [6].

(1) Definition of *belief structure*

Let L be a language. The total information that an agent A has at time t is called the *belief universe* of A at t and denoted by *Belief-Universe*(A, t). The *belief structure* of A at time t, denoted by *Belief-Structure*(A, t), is a two-sorted structure S, {0, 1/2, 1} , >, ~, , ≥, *eval* that is defined as follows:

i) S is a set of sets of sentences in L, and S = *Belief-Universe*(A, t).

ii) > is a partial ordering on S^2.

iii) ~ is an equivalence relation on S^2.

iv) 1 ≥ 1 & 1/2 ≥ 1/2 & 0 ≥ 0 & 1 ≥ 1/2 & 1 ≥ 0 & 1/2 ≥ 0.

v) X_1, X_2, X_3, X_4 S ($X_1 \sim X_2$ & $X_3 \sim X_4$ & $X_1 > X_3$ $X_2 > X_4$).

vi) X S (*eval*(X) = 1 *or* *eval*(X) = 1/2 *or* *eval*(X) = 0).

vii) X, Y S ($X > Y$ *eval*(X) ≥ *eval*(Y)).

viii) X, Y S ($X \sim Y$ *eval*(X) = *eval*(Y)).

ix) X, Y S (X Y & *eval*(Y) = 1 *eval*(X) = 1).

x) X, Y S (X Y & *eval*(X) = 0 *eval*(Y) = 0).

It can be easily shown that this definition is sound.

(2) Proposition 1

Definition (1) is sound, i.e., there exists a structure that satisfies it.

Proof. It suffices to give a structure that satisfies all conditions in (1). Suppose *S* = {*A*, *B*, *C*, *D*, *E*, *F*}, *A* = {*p*, *q*}, *B* = {*q*}, *C* = {*r*}, *D* = {*s*}, *E* = {*t*, *u*}, and *F* = {*t*}. Suppose that conditions from (1.i) to (1.iv) are satisfied. Furthermore, suppose:

$A > C > E$ &
$B > D > F$ &
$A > D > E$ &
$B > C > F$ &
$A \sim B$ & $C \sim D$ & $E \sim F$ &
eval(*A*) = *eval*(*B*) = 1 &
eval(*C*) = *eval*(*D*) = 1/2 &
eval(*E*) = *eval*(*F*) = 0.

It is easy to check that all conditions from (1.v) to (1.x) are satisfied by this stipulation. Thus there exists a structure that satisfies definition (1).

Based on definition (1), *belief system* and *possibility system* can be defined.

(3) Definition of *belief system* and *possibility system*

i) The *belief system* of *A* at *t*, denoted by *Belief-System*(*A*, *t*), is defined as follows:

Belief-System(*A*, *t*) =
{*X*: *X* *Belief-Universe*(*A*, *t*) & *eval*(*X*) = 1}.

ii) The *possibility system* of *A* at *t*, denoted by *Possible*(*A*, *t*), is defined as follows:

Possible(*A*, *t*) =
{*X*: *X* *Belief-Universe*(*A*, *t*) & *eval*(*X*) ≥ 1/2}.

According to (3), an agent *A* believes any sentence in his belief system to be true and any sentence in *Possible*(*A*, *t*) to be possibly true. This definition allows cases where belief systems are inconsistent or not deductively closed. It is well-known that a deductively closed set of sentences is infinite and rejects finite representation (cf. [8]). Even a belief structure that creates an inconsistent belief system can have parts that are sound and useful for several problem-solving activities. Such a belief structure faces difficulties where inconsistency becomes apparent, but it might be useful in solving small problems to which only a consistent part of the belief system is applied.

An important feature of belief structures is expressed in the following corollary. The corollary expresses that a high ranked set of sentences is more firmly believed than a low ranked one.

(4) Corollary 1

i) *X*, *Y* *S* (*X* > *Y* & *Y* *Belief-System*(*A*, *t*) *X* *Belief-System*(*A*, *t*)).

ii) *X*, *Y* *S* (*X* > *Y* & *Y* *Possible*(*A*, *t*) *X* *Possible*(*A*, *t*)).

Proof. Suppose *X* > *Y* & *Y* *Belief-System*(*A*, *t*). Then, from (3.i), *eval*(*Y*) = 1. From (1.iv), (1.vi), and (1.vii), *eval*(*X*) = 1. Thus from (3.i), *X* *Belief-System*(*A*, *t*). From this follows (4.i). (4.ii) can be analogously proved from (1.iv), (1.vi), (1.vii), and (3.ii).

TBS can be applied to descriptions of belief revisions related to scientific activities. For scientific reasoning, it will be appropriate to require a discipline that is more strict than usual. The following two principles express a proposal for the characterization of scientific reasoning.

(5) Principles for intelligent agents

i) An intelligent agent desires that his belief system is consistent. Therefore, he tries to resolve any contradiction when he finds one.

ii) An intelligent agent desires that his belief system becomes richer in the long run.

These principles are fundamental because they can influence agents' behavior. (5.i) is obviously important for the formulation of a theory. (5.ii) becomes important for a theory choice. According to (5.ii), an intelligent agent is *progressive* rather than *conservative*. This means that (5.ii) expresses Laudan's view of science rather than Quine's (cf. [5], [7]). A progressive agent often keeps tradition, because he does not want to lose successful results that he has already achieved. However, his aim is not to preserve tradition but to make further progress. Thus he is ready to radically modify the traditional view, when he thinks that it makes his belief system richer.

Many authors, for example Kuhn, Lakatos, and Laudan, pointed out that there is no definitive formal criterion for theory choice (cf. [3]. [4], [5]). Therefore, principle (5.ii) is a deliberately vague formulation. Each scientist decides which theory is better and performs his work according to this decision. Not a formal criterion but history decides which theory has been more effective for predictions and explanations. However, it is also a fact that sciences have created theories that are more precise and more expressive than earlier ones. In section 4, I will discuss this subject in more detail.

There are six methods for alteration of a belief structure:

1. change of evaluations,
2. change of ordering,
3. addition of a new piece of information,
4. deletion of an existing piece of information,
5. inference, and
6. expansion of language.

When after some considerations agent A comes to be convinced that T is true, A's evaluation of T might be changed from 1/2 to 1. A's change of belief ordering occurs when A changes his opinion about which ideas are fundamental. Perception and reception of new information add new sentences to the given belief structure. Deletion corresponds to forgetting a belief. Thus it is appropriate to express something that has become unbelievable in TBS not by its deletion but in terms of the reduction of its value. Inference can create a new belief. *Expansion of language* is related with *scientific revolution*, which is the subject of the fourth section of this paper. By combining these six elementary operations, complex changes of belief structures can be expressed.

It is now possible to characterize some ordinary expressions of epistemic states by using TBS:

(6) Characterizations of epistemic states within TBS

i) A believes that X is true iff $eval(X) = 1$ with respect to A's belief structure.

ii) A believes that X is false iff $eval(X) = 0$ with respect to A's belief structure.

iii) A believes that X might be true iff $eval(X) = 1/2$ with respect to A's belief structure.

iv) A starts doubting a statement X iff X's ranking in A's belief structure diminishes.

v) A believes that X is absolutely right iff X is high ranked in A's belief structure.

vi) If A is not sure which of X and Y is more preferable, then neither $X > Y$ nor $Y > X$.

vii) If A believes that X is more fundamental than Y, then A holds the relation $X > Y$ during his belief change.

By using this belief ordering, some normative requirements for scientists can be expressed:

(7) Normative requirements for scientists

i) If A believes that X is an *auxiliary hypothesis* of Y, then A should believe that Y is more fundamental than X.

ii) If A infers Y from $X_1,\ldots, X_n$, then A should believe that each of $X_1,\ldots, X_n$ is at least as fundamental as Y.

iii) If A infers Y from $X_1,\ldots, X_n$ and $eval(X_k) = 1$ for all k with $1 \leq k \leq n$, then A should believe that Y is true.

iv) If X is a scientific observation, then there is a theory T such that A should believe that T is more fundamental than X. This is a consequence from the *theory ladeness of observation*.

(6.vii) and (7.i) support Lakatos' thesis that a fundamental theory can be protected by auxiliary hypotheses (cf. [4]). When a contradiction with an observation is discovered, some auxiliary hypotheses are replaced without change of the fundamental theory. This occurs because the fundamental theory is more firmly believed than its auxiliary hypotheses. The last statement follows from (6.vii) and (7.i). Thus a fundamental theory is rejected, only if a competing fundamental theory is proposed and gains full confidence.

2 Problems associated with the AGM Theory

The AGM theory of belief revision is a solid formal framework that has been extensively studied (cf. [1], [2]). However, there are some shortcomings of the AGM theory, since it works with belief sets and the associated formal constraints are too strong when *epistemic entrenchment* is involved. TBS shows a different possibility of a formal treatment of theory change, even if its research is still immature.

The AGM theory defines an epistemic entrenchment ordering for a belief state K by the following axioms (cf. [1], [8]):

E1: If $\varphi \leq_K \psi$ and $\psi \leq_K \chi$ then $\psi \leq_K \chi$ (*transitivity*);

E2: If ψ follows from φ then $\varphi \leq_K \psi$ (*dominance*);

E3: Either $\varphi \leq_K \varphi\ \ \psi$ or $\psi \leq_K \varphi\ \ \psi$ (*conjunctiveness*);

E4: If K is consistent then $\varphi \leq_K \psi$ for all ψ iff not ($\varphi\ \ K$) (*minimality*);

E5: If $\varphi \leq_K \psi$ for all φ, then ψ is a theorem (*maximality*);

Axioms E1-E3 imply that $\leq_K$ is a linear order (cf. [8]). This means that any two sentences are comparable and that it can be asked which of them is at least as epistempologically entrenched as another. It turns out that these constraints are quite restrictive and make many ideas inexpressible.

The most fundamental problem with the AGM theory is that it is primarily a theory based on set representation and not based on structural representation. Operations for belief change are defined only with respect to belief sets. As a result, structural information given by an epistemic entrenchment ordering disappears after operations of belief change.

The second problem is that the AGM theory admits only a minimal belief change, as Gardenfors and Rott characterized it:

> “The amount of information lost in a belief change should be kept minimal.” ([2] p.38)

This restriction is obviously too strong to describe belief changes in sciences, because there are not only minimal but also radical belief changes in sciences (cf. [3], [4]).

TBS shows that even by a pure rethinking a belief system can be radically changed. Suppose that T_1 and T_2 be two competing theories, Aux_1 be an auxiliary hypothesis for T_1, and Aux_2 be an auxiliary hypothesis for T_2. Furthermore, suppose that scientist A believes T_1 and not T_2 and that he holds at t_1:

$T_1 > Aux_1 > T_2 > Aux_2 >$ &
$eval(Aux_1) = 1$ &
$eval\ (T_2) = 0$.

At t_2, after some thought, A changes his mind and he comes to believe:

$T_2 > Aux_2 > T_1 > Aux_1 >$ & $eval(Aux_2) = 1$ & $eval\ (T_1) = 0$.

This results in a radical change of his belief system, namely it holds:

$T_1 \quad Aux_1 \quad Belief\text{-}System(A, t_1)$ &
$not\ (T_2 \quad Belief\text{-}System(A, t_1))$ &
$T_2 \quad Aux_2 \quad Belief\text{-}System(A, t_2)$ &
$not\ (T_1 \quad Belief\text{-}System(A, t_2))$.

A perspective change of a scientist who has accepted a new theory might be explained in this way. At first, the scientist, who was acquainted with these theories, was occupied by the older view provided by T_1. Later, he comes to accept theory T_2 and has to reorganize his belief structure, so that T_2 and its auxiliary hypotheses become dominant. This reorganization causes his perspective change.

The arguments against the AGM theory that have been brought in this section can be also applied to many of its variations.

3 Examples of Belief Structures

In this section, parts of some belief structures are described. Examples will show that research traditions are influential even during a period of paradigm change.

In the fourth century B.C., Greek scientists believed in *Greek astronomy* (*GA*) whose fundamental theses can be reconstructed as follows:

(8) *Greek astronomy*:

$GA = \{g_1, g_2, g_3, g_4, g_5, g_6, g_7\}$.

- g_1: The earth is spherical.
- g_2: The entire universe is contained within a stellar sphere.
- g_3: The universe is spherical.
- g_4: The earth is located in the center of the universe.
- g_5: There are exactly seven planets, namely the moon, Mercury, Venus, the sun, Mars, Jupiter, and Saturn.
- g_6: All planets revolve round the earth.
- g_7: The stellar sphere revolves on an axis from East to West.

Copernicus, i.e. Niklas Koppernigk (1473-1543), rejected some important *GA* theses, but there are also *GA* theses that he took for granted.

(9) New theses of Copernicus:

$Cop := \{c_1, c_2, c_3, c_4, c_5\}$.

- c_1: The sun is located at the center of the universe.
- c_2: The earth revolves round the sun.
- c_3: The earth revolves on its axis from East to West.
- c_4: The moon revolves round the earth.
- c_5: Mercury, Venus, Mars, Jupiter, and Saturn revolve round the sun.

Copernican astronomy (*CA*) consists of *Cop* and a part of *GA*:

$CA := Cop \quad \{g_1, g_2, g_3\}$.

His belief structure satisfies, therefore, the following condition:

$eval(Cop) = 1$ &

$eval(\{g_1, g_2, g_3\}) = 1$ &
$eval(\{g_4, g_5, g_6, g_7\}) = 0$.

With respect to belief ordering, there is some interpretation freedom. When we think that he did not have any doubt about g_1, g_2, and g_3, the following interpretation is appropriate:

$$\{g_1, g_2, g_3\} > Cop > \{g_4, g_5, g_6, g_7\}.$$

Tycho Brahe (1546-1601) proposed an astronomy that is almost in harmony with the traditional view. He modified *CA*, so that it became compatible with a large part of *GA*.

(10) Brahe's thesis

t_1: The circle of the sun is centered on the earth.

Tychonic cosmology (*TC*) consists of t_1, a part of *Cop*, and a part of *GA*.

$$TC := \{t_1\} \quad \{c_4, c_5\} \quad \{g_1, g_2, g_3, g_4, g_5, g_7\}.$$

Thus his belief structure satisfies the following condition:

$eval(\{ t_1\}) = 1$ &
$eval(\{g_1, g_2, g_3\}) = 1$ &
$eval(\{g_4, g_5, g_7\}) = 1$ &
$eval(\{c_4, c_5\}) = 1$,
$eval(\{g_6\}) = 0$ &
$eval(\{c_1, c_2, c_3\}) = 0$.

A possible interpretation of his belief ordering is the following:

$\{g_1, g_2, g_3\} > \{t_1\} > \{g_4, g_5, g_7\} >$
$\{c_4, c_5\} > \{g_6\}$ &
$\{c_4, c_5\} > \{c_1, c_2, c_3\}$.

This interpretation is supported by the fact that no one doubted g_1, g_2, and g_3 at that time.

GA, *CA*, and *TC* are incompatible, but we can still find continuity among belief structures of people who believed one of these theories. Contrarily, the fact that two scientists have the same belief system does not guarantee the same behavior with respect to theory change. For example, there might exist a Copernican with

$$\{g_1, g_2, g_3\} > Cop \ \& \ eval(Cop) = 1$$

and a Copernican with

$\{g_1\} > Cop(-) > \{c_1, g_2, g_3\}$ &
$eval(\{c_1, g_2, g_3\}) = 1$,

where

$$Cop(-) := Cop - \{c_1\}.$$

Only the latter is similar to the contemporary view, which can be characterized as follows:

$\{g_1\} > Cop(-) > \{c_1, g_2, g_3\}$ &
$eval(Cop(-)) = 1$ &
$eval(\{c_1, g_2, g_3\}) = 0$.

These examples also show how important it is to describe a set of sentences as a unit. Identifying a set of assumptions that might be false is a fundamental process for theory change.

4 Scientific Revolutions and Expansion of Language

A belief structure can be dramatically changed by an expansion of language. In section 1, six methods for alteration of a belief structure are mentioned. Change of evaluations, change of ordering, and inference do not change the belief universe. Addition of a new piece of information expands the belief universe, whereas deletion of an existing piece of information reduces it. Expansion of language is the most radical alteration and it usually introduces a continuing process of expansion of belief universe.

A language is expanded when an acquaintance with a new theory introduces new terminologies. When this new theory is accepted, it will be applied to predictions and explanations. It will sometimes provide new explanations for old phenomena. As a result, a radical adjustment of belief structure will be performed. This change might be characterized as *revolutionary*. A *scientific revolution* is nothing but such a radical belief change that is performed in the entire *scientific community*.

In the history of science, some scientific revolutions have introduced new mathematical theories and have provided their new application areas. For example, Newtonian dynamics introduced differential calculus, while the relativity theory provided an application area for non-Euclidean geometry.

Conceptual expansion is also a form of expansion of language. It is realized by introduction of a new theory that uses new terminologies. Conceptual expansion is common also in non-scientific disciplines. Ideas that are expressed in the expanded language are sometimes not expressible in the old language and they are not intelligible to old-fashioned scientists. Expansion of language and change of language is a source of incommensurability. Usually, expansion of language makes a belief system richer, because it makes more beliefs expressible. It is, therefore, a method to fulfill the desire mentioned in (5.ii).

As Kuhn explicated, a scientific revolution is a change of beliefs and activities in a scientific community (cf. [7]). It requires that a radically new belief structure of a scientist comes to be shared by the majority of scientists in the related discipline. An expanded part of language introduces a new ontology and metaphysics, and it sometimes influences the interpretation of the old part of language.

5 Conclusions

In this paper I proposed a new theory of belief description, TBS. TBS describes a belief state in terms of a structure and directly manipulates belief structures. A change of belief systems results from a change of belief structures. Because of its weak constraints, its formal consequences are poor but it is widely applicable to descriptions of belief change. Furthermore, TBS is compatible with the progressive view of science proposed by Laudan. In fact, TBS can be used to describe different kinds of epistemic principles. The use of the framework was demonstrated by applying TBS to changes in astronomical theory.

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